(مائل) oblique incidence

- At normal incident we assumed that the wave travelling at specific direction (one direction)

- General equation assuming we deal with lossless medium.

$$E(r,t) = E_o \cos(k.r - wt).$$

$$= \mathrm{Re}\left[E_o e^{j(k.r - wt)}\right.$$

- where;

$r = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ position vector

$k = k_x\hat{a}_x + k_y\hat{a}_y + k_z\hat{a}_z$ wave number vector

- $k.r = k_x.x + k_y.y + k_z.z$ (constant)

- $k^2 = k_x^2 + k_y^2 + k_z^2 = w^2\mu\epsilon$

$$\therefore \sqrt{k^2} = \sqrt{w^2\mu\epsilon} = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

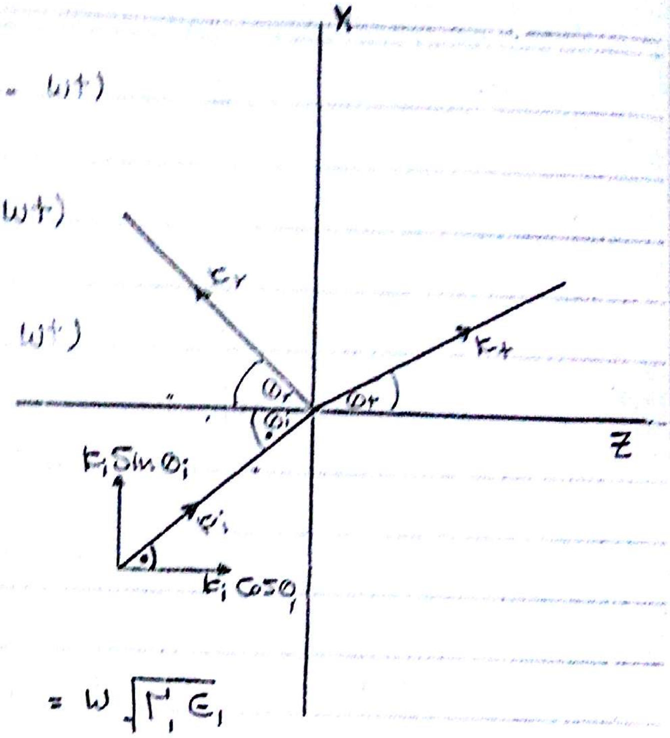
$$\therefore |k| = w\sqrt{\mu\epsilon}$$

$$|k| = \beta$$

$E_i = E_{i0} \cos(k_{ix} x + k_{iy} y + k_{iz} z - \omega t)$

$E_r = E_{r0} \cos(k_{rx} x + k_{ry} y + k_{rz} z - \omega t)$

$E_t = E_{t0} \cos(k_{tx} x + k_{ty} y + k_{tz} z - \omega t)$



Assume

① $k_i = \beta_1$

$k_r = \beta_1$ $\quad\therefore k_i = k_r = \beta_1 = \omega\sqrt{\mu_1 \epsilon_1}$

② $k_t = \beta_2$ $\quad\therefore k_t = \beta_2 = \omega\sqrt{\mu_2 \epsilon_2}$

At The interface ($z = 0$)

Boundary Conditions $\quad E_{i0} + E_{r0} = E_{t0}$

* $\omega_i = \omega_r = \omega_t = \omega$ → Frequency is unchanged

$k_{ix} = k_{rx} = k_{tx} = k_x$

$k_{iy} = k_{ry} = k_{ty} = k_y$

$k_{iz} = k_{rz} = k_{tz} = k_z$

→ apply phase matching condition (means that tangential components of propagation vectors be continous).

$k_i \sin\theta_i = k_r \sin\theta_r$

$k_i \sin\theta_i = k_t \sin\theta_t$

- For lossless medium

① $K_i = K_r$

$\theta_i = \theta_r$ "reflection law"

② $K_i \sin\theta_i = K_t \sin\theta_t$

$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$

$\omega\sqrt{\mu_1\epsilon_1} \sin\theta_i = \omega\sqrt{\mu_2\epsilon_2} \sin\theta_t$ $(* c)$

$c\sqrt{\mu_1\epsilon_1} \sin\theta_i = c\sqrt{\mu_2\epsilon_2} \sin\theta_t$

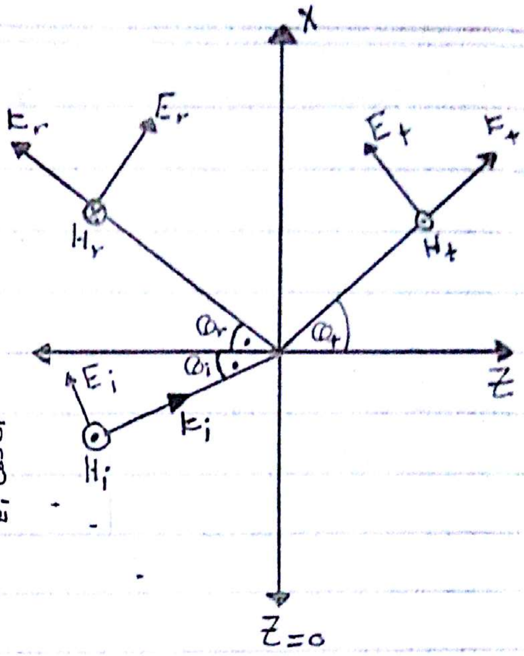
"Snell's law"

* $n_1 \sin\theta_i = n_2 \sin\theta_t$

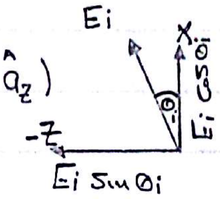
$n_1 = c\sqrt{\mu_1\epsilon_1}$ & $n_2 = c\sqrt{\mu_2\epsilon_2}$

* parallel polarization

(E is parallel to the plane of incidence)



. E at xz plane



① $E_{is} = (E_{io}\cos\theta_i\, \hat{a}_x - E_{io}\sin\theta_i\, \hat{a}_z) \cdot e^{-j k_i \cdot r}$

$= E_{io}(\cos\theta_i\, \hat{a}_x - \sin\theta_i\, \hat{a}_z) \cdot e^{-j k_i \cdot r}$

→ $k_i = k_i \sin\theta_i\, \hat{a}_x + k_i \cos\theta_i\, \hat{a}_z$

∴ $E_{is} = E_{io}(\cos\theta_i\, \hat{a}_x - \sin\theta_i\, \hat{a}_z)\, e^{-j(k_i \sin\theta_i \cdot x + k_i \cos\theta_i \cdot z)}$

$k_i = \beta_1$

$= E_{io}(\cos\theta_i\, \hat{a}_x - \sin\theta_i\, \hat{a}_z)\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$

→ $H_{is} = \frac{E_{io}}{\eta_1} \cdot e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\, \hat{a}_y$

→ apply phase matching Condition (E & H to be continuous)

→ $E_1 = E_2$. at $z = 0$

$$E_{io} \cos\theta_i + E_{ro} \cos\theta_r = E_{to} \cos\theta_t$$

$$(E_{io} + E_{ro}) \cos\theta_i = E_{To} \cos\theta_t \longrightarrow ①$$

→ $H_1 = H_2$

$$H_{io} + H_{ro} = H_{to}$$

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2} E_{to}$$

$$\rightarrow \Gamma_{11} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\rightarrow T_{11} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\rightarrow 1 + \Gamma_{11} = T_{11} \left(\frac{\cos\theta_t}{\cos\theta_i} \right)$$

$$\rightarrow \cos\theta_t = \sqrt{1 - \left(\frac{u_2}{u_1} \sin\theta_i\right)^2}$$

→ Brewster angle (polarizing angle) $\theta_{B_{\parallel}}$

is the incidence angle which make reflection co.eff = 0

. There is no reflection.

→ $$\Gamma_{\parallel} = 0 = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\therefore \eta_2 \cos\theta_t = \eta_1 \cos\theta_i \qquad (\theta_i = \theta_{B_{\parallel}})$$

$$\eta_2^2 \cos^2\theta_t = \eta_1^2 \cos^2\theta_{B_{\parallel}}$$

$$\eta_2^2 (1 - \sin^2\theta_t) = \eta_1^2 (1 - \sin^2\theta_{B_{\parallel}})$$

$$\therefore \sin^2\theta_{B_{\parallel}} = \frac{\eta_2^2}{\eta_1^2}(1 - \sin^2\theta_t) - 1$$

$(n_1 \sin\theta_B = n_2 \sin\theta_t)$

$$\therefore \sin^2\theta_{B_{\parallel}} = \frac{1 - (\mu_2/\epsilon_2)/(\mu_1/\epsilon_1)}{1 - (\epsilon_1/\epsilon_2)^2}$$

$$= \frac{1 - \mu_2\epsilon_1/\mu_1\epsilon_2}{1 - (\epsilon_1/\epsilon_2)^2}$$

For nonmagnetic mediums

$$\mu_1 = \mu_2 = \mu_0$$

$$\sin\theta_{B_{\parallel}} = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}} \quad \& \quad \tan\theta_{B_{\parallel}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1}$$

→ tangential Components of E and H to be Continous at $z=0$

$(\theta_i = \theta_r)$

↳ $E_1 = E_2$

$$E_{io} + E_{ro} = E_{to} \quad \rightarrow (1)$$

and

↳ $H_1 = H_2$

$$\frac{\cos\theta_i}{\eta_1}(E_{io} - E_{ro}) = \frac{\cos\theta_t}{\eta_2} E_{to} \quad \rightarrow (2)$$

From (1) and (2)

$$\Gamma_{\perp} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_{\perp} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

↳ $1 + \Gamma_{\perp} = \tau_{\perp}$

Brewster Angle (Total transmission)

① $T_\perp = 1$ & $\Gamma_\perp = 0$ (no reflection)

putting $\Gamma_\perp = 0$

$$\eta_2 \cos\theta_{B\perp} = \eta_1 \cos\theta_t$$

$$\eta_2^2 \cos^2\theta_{B\perp} = \eta_1^2 \cos^2\theta_t$$

$$\eta_2^2 (1 - \sin^2\theta_{B\perp}) = \eta_1^2 (1 - \sin^2\theta_t)$$

$$\therefore \sin^2\theta_{B\perp} = \frac{1 - (\mu_1\epsilon_2/\mu_2\epsilon_1)}{1 - (\mu_1/\mu_2)^2}$$

① For non magnetic $\mu_1 = \mu_2 = \mu_0$

$$\sin^2\theta_{B\perp} = \infty \quad \text{(means } \theta_{B\perp} \text{ not exist)}$$

② For $\epsilon_1 = \epsilon_2$

$$\sin\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1 + \mu_2}}$$

$$\tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$$

A uniform plane wave in Air with $(8\cos(\omega t - 4x - 3z)\hat{a}_y$ V/m

is incident on a dielectric slap $(z \geq 0)$

with $\mu_r = 1$, $\epsilon_r = 2.5$; $\sigma = 0$. Find.

a) The polarization of the wave
b) The angle of incidence
c) The reflected component
d) The transmitted component

Solution :-

a) $E = 8\cos(\omega t - 4x - 3z)\ \hat{a}_y$

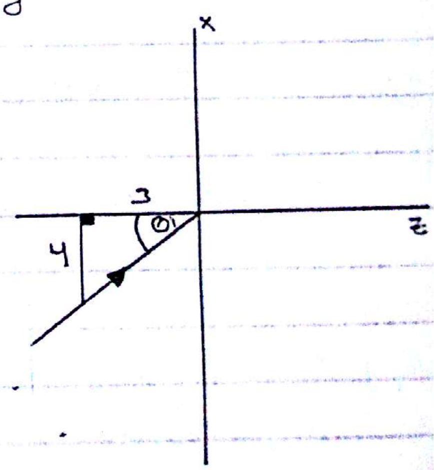
The wave travels in $(x - z)$ plane and E is perpendicular
(we have perpendicular polarization)

b) $E_{is} = 8\cos(\omega t - k_i \cdot r)\ \hat{a}_y$

$k_{ix} = 4$, $k_{iz} = 3$

$\tan\theta_i = \frac{4}{3}$

$\theta_i = 53.13^\circ$



c)

$$E_{rs} = E_{r_0} \cos(\omega t - k_r . r) \hat{a}_y$$

$$k_i = k_r$$

$$E_{r_0} = \Gamma_{\perp} . E_{i_0}$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\eta_1 = 120\pi$$

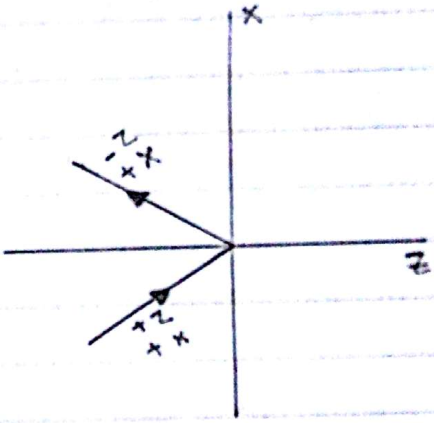
$$\eta_2 = \frac{120\pi}{\sqrt{2.5}}$$

$$\theta_i = 53.13°$$

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\cancel{c}\sqrt{\mu_1 \epsilon_1} \sin 53.13 = \cancel{c}\sqrt{\mu_2 \epsilon_2} \sin\theta_t$$

$$\therefore \theta_t = 30.39°$$



$$\therefore \Gamma_{\perp} = -.389$$

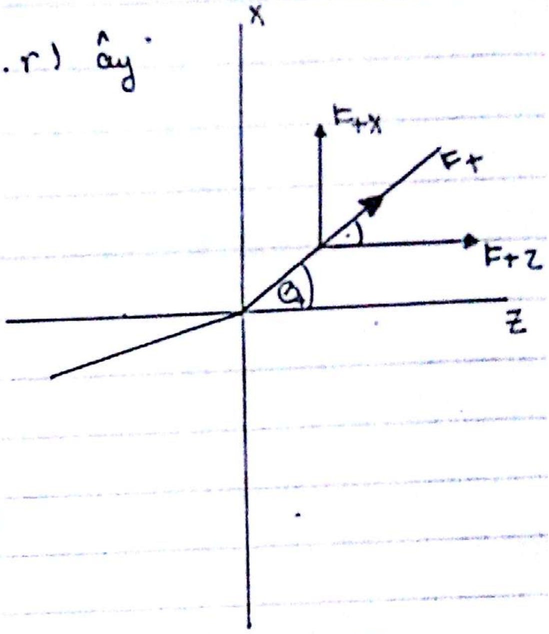
$$\therefore E_{rs} = -.389 * 8 \cos(\omega t - 4x + 3z) \hat{a}_y$$

$$= -3.112 \cos(\omega t - 4x + 3z) \hat{a}_y$$

Scanned by CamScanner

d) $E_{ts} = E_{to} \cos(\omega t - k_t . r)\, \hat{a}_y$

$k_t = k_{tx}\, \hat{a}_x + k_{tz}\, \hat{a}_z$



$\therefore \beta = |k|$

$\beta_2 = |k_t|$

$\omega \sqrt{\mu_2 \epsilon_2} = |k_t|$

$15 * 10^8 \sqrt{\mu_o \mu_r \epsilon_o \epsilon_r} = |k_t|$

$\beta_1 = |k_i| = 5$

$\therefore |k_t| = 7.906$

$\therefore 5 = \omega \sqrt{\mu_o \epsilon_o}$

$\therefore \omega = 15 * 10^8$ rad/s

$\rightarrow k_{tx} = 7.906 \sin\theta_t$

$= 4$

$\rightarrow k_{tz} = 7.906 \cos\theta_t$

$= 6.819$

$\therefore k_t = 4\, \hat{a}_x + 6.819\, \hat{a}_z$

$E_{to} = \tau_\perp E_{io}$

$$\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = .611$$

ا أهم حاجه التخرج

$$E_{to} = .611 * 8 = 4.888$$

$$\therefore E_t = 4.888 \cos(15*10^8 t - 4x - 6.819z)\ \hat{a}_y$$

$$\text{①}\quad E_i = 8\cos(15*10^8 t - \overset{+a_x}{4x} - \overset{+a_z}{3z})\ \hat{a}_y$$

$$H_i = \frac{\hat{a}_k \times E}{\eta_1} \qquad \hat{a}_k = \frac{4\hat{a}_x + 3\hat{a}_z}{5}$$

$$= \frac{(4\hat{a}_x + 3a_z) \times 8\hat{a}_y}{5 * 120\pi} \cdot \cos(15*10^8 t - 4x - 3z)$$

$$= \left(\frac{4\times 8}{5*120\pi}\hat{a}_z - \frac{3\times 8}{5*120\pi}\hat{a}_x\right) \cdot \cos(\quad)$$

$$= (\underset{-.01273}{-.012}\,\hat{a}_x + \underset{.01697}{.016}\,\hat{a}_z) \cdot \cos(\omega t - 4x - 3z)\ \text{A/m}$$

②

$$E_r = -3.112 \cos(\omega t - 4x + 3z)\ \hat{a}_y$$

$$H_r = (\underset{.012 * \Gamma_I}{-4.776}\,\hat{a}_x - \underset{.016 * \Gamma_I}{6.368}\,\hat{a}_z) \cdot \cos(\omega t - 4x + 3z)$$

$\underline{mA/m}$

k	$+a_x$	$-a_z$
E	$-a_y$	$-a_y$
H	$-a_z$	$-a_x$

Scanned by CamScanner

③ $E_t = 4.888 \cos(wt - 4x - 6.819z)\ \hat{a}_y$

$\therefore H_t = (-17.69\ \hat{a}_x + 6.37\ a_z) \cdot \cos(wt - 4x - 6.819z)$ mA/m

Scanned by CamScanner

An EMW with $E = (10\,\hat{a}_y + 5\,\hat{a}_z).\cos(\omega t + 2y - 4z)$ V/m in free space and incident on a dielectric with ($\mu_r = 1$; $\epsilon_r = 4$; $\sigma = 0$) at ($z \geq 0$). with a transmition angle $= 30^\circ$

a) ω and λ

b) Find all other Components

c) Time average power

→ Solution ←

①

$$E_i = (10\,\hat{a}_y + 5\,\hat{a}_z).\cos(\omega t + 2y - 4z) \quad V/m$$

$$\therefore k_i = k_{ix}\,\hat{a}_x + k_{iy}\,\hat{a}_y + k_{iz}\,\hat{a}_z$$

$$= -2\,\hat{a}_y + 4\,\hat{a}_z$$

$$\therefore \beta_1 = |k_i|$$

$$= \sqrt{2^2 + 4^2} = 2\sqrt{5}$$

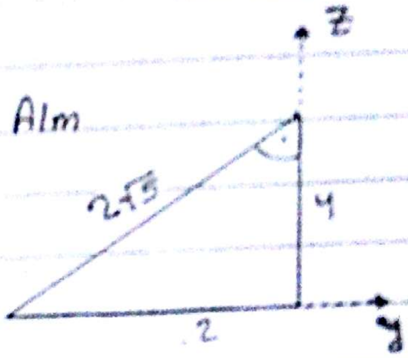
where $\beta = \frac{\omega}{c}$ at free space

$$\therefore \omega = \beta . c = 1.342 \times 10^9 \text{ rad/s}$$

and so ; $\beta = \frac{2\pi}{\lambda}$

$$\therefore \lambda = 2\pi/\beta = 1.405 \text{ m}$$

The wave is parallel polarized



$$H_i = \frac{E_{io}}{\eta_1} \cos(\omega t + 2y - 4z)\ \hat{a}_x \quad A/m$$

حط قيمها

$$= \frac{5\sqrt{5}}{120\pi} \cos(\omega t + 2y - 4z)$$

$$= -.029656 \cos(1.342 \times 10^{8} t + 2y - 4z)\ \hat{a}_x \quad A/m$$

- $E_{ro} = \Gamma_{11} \cdot E_{io}$
- $E_{to} = \tau_{11} \cdot E_{io}$
- $H_{ro} = \Gamma_{11}\ H_{io}$
- $H_{to} = \tau_{11}\ H_{io}$

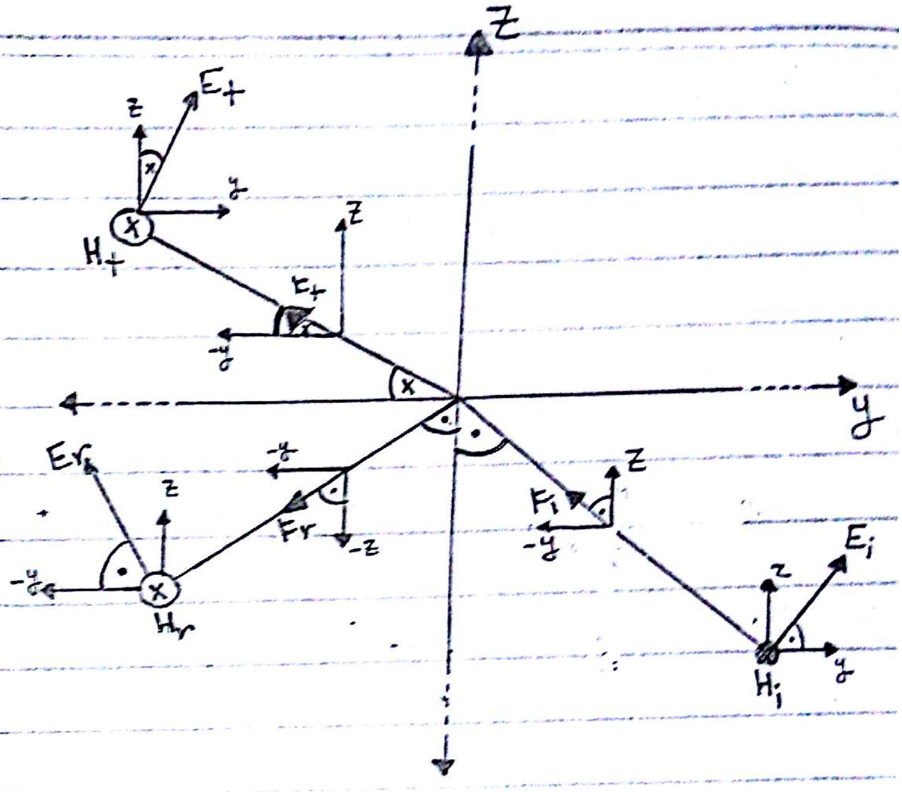
$$\Gamma_{11} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$\eta_1 = 120\pi$ $\qquad \eta_2 = \sqrt{\frac{\mu}{\epsilon}} = 60\pi$ $\qquad \theta_i = \tan^{-1}\frac{2}{4} = 26.56^\circ$

$$\Gamma_{11} = \frac{60\pi \cos 30 - 120\pi \cos 26.56}{60\pi \cos 30 + 120\pi \cos 26.56} = -.347$$

$$\tau_{11} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = .6738$$

$E_{ro} = -3.879$



$E_r = (-3.47\ \hat{a}_y + 1.73\ \hat{a}_z)$

$\cdot \cos(1.342 * 10^9 t + \ldots\ldots\ldots)$

$|k_i| = |k_r| = \beta_1$

$k_r = 2\ \hat{a}_y - 4\ \hat{a}_z$

$E_r = (-3.47\ \hat{a}_y + 1.73\ \hat{a}_z) \cdot \cos(1.342 * 10^9 t + 2y + 4z)$ V/m

Example 3-

A polarized wave is incident from Air to polystrene

With $\mu = \mu_0$; $\epsilon = 2.6\ \epsilon$ at Brewster Angle.

Determine the Transmission angle ?

→ Solution ←

① polarized → parallel, → perpendicular

② $\mu = \mu_0$ → non magnetic material

$\therefore \theta_{B\perp}$ not Exist For non magnetic

$$\therefore \tan \theta_{B_{11}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = 1.612$$

$$\therefore \theta_{B_{11}} = 58.19^\circ$$

$$n_1 \sin \theta_{B_{11}} = n_2 \sin \theta_t$$

$$w\sqrt{\mu_1 \epsilon_1} \sin \theta_{B_{11}} = w\sqrt{\mu_2 \epsilon_2} \sin \theta_t$$

$$\therefore \theta_t = 31.8^\circ$$

Scanned by CamScanner

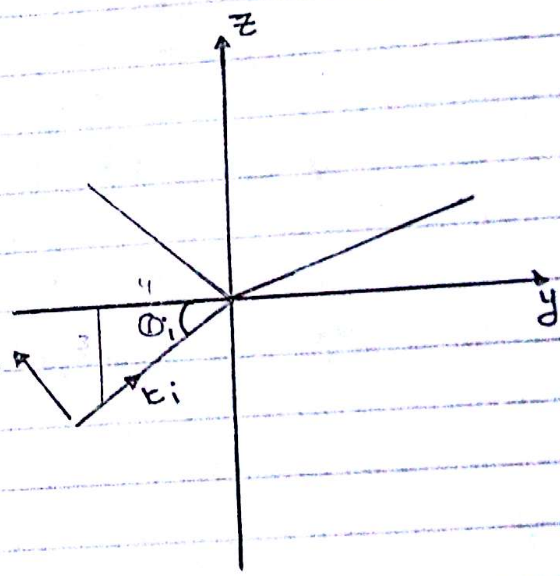
polarized wave in Air with $E = (8\hat{a}_y - 6\hat{a}_z) \sin(\omega t - 4y - 3z)$ V/m

incident on a dielectric half-space with ($\epsilon = 4\epsilon_0$; $\mu = \mu_0$)
at ($y \geq 0$)

Find :-

1. The incidence angle
2. The time average in Air ($\mu = \mu_0$; $\epsilon = \epsilon_0$)
3. The reflected and transmitted E

→ Solution ←



① $k_i = 4\hat{a}_y + 3\hat{a}_z$

$\tan\theta_i = \frac{3}{4}$

$\therefore \theta_i = 36.87^\circ$

② $P_{av} = \frac{1}{2} Re(E \times H^*)$

$= \frac{1}{2} \frac{E_0^2}{\eta_1} \hat{a}_k$

$= \frac{(\sqrt{8^2+6^2})^2}{2 * 120\pi} \cdot \frac{4\hat{a}_y + 3\hat{a}_z}{5}$

$= 79.58\,\hat{a}_y + 106.1\,\hat{a}_z$

$\theta_i = \theta_r = 36.87^\circ$

$E_r = (E_{ry}\,\hat{a}_y + E_{rz}\,\hat{a}_z)\ \sin(wt - k_r \cdot r)$

$|k_i| = |k_r| = 5$

$\cdot\ k_r = k_{rz}\,\hat{a}_z - k_{ry}\,\hat{a}_y$

$= 3\,\hat{a}_z - 4\,\hat{a}_y$

now we need $|E_{ro}|$.

$|E_{ro}| = \Gamma_{11} \cdot |E_{io}|$

$$\Gamma_{11} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$\theta_t = ???$

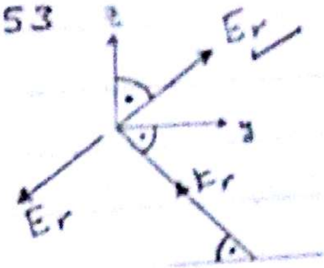
$- n_1 \sin\theta_i = n_2 \sin\theta_t$

$\therefore\ \theta_t = 17.4^\circ$

$\therefore\ \Gamma_{11} = -.253$

also $\tau_{11} = .6265$

$|E_{r0}| = 10 * (-.253) = -2.53$



$E_r = (1.518\ \hat{a}_y + 2.024\ \hat{a}_z)$

$\sin(wt + 4y - 3z)$ v/m

to get E_t

$E_t = (E_{ty}\ \hat{a}_y + E_{tz}\ \hat{a}_z) . \sin(wt - k_t . r)$

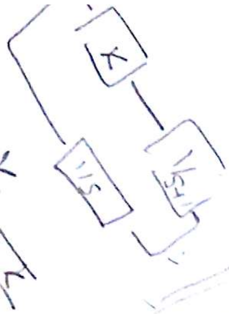
$|k_t| = \beta_2 = w\sqrt{\mu_2 \epsilon_2}$

$|k_t| = 10$

$\therefore k_t = 9.539\ \hat{a}_y + 3\ \hat{a}_z$

$|E_{t0}| = \tau_{11}\ |E_{i0}| = .265$

$\therefore E_t = (1.877\ \hat{a}_y - 5.968\ \hat{a}_z) \sin(wt - 9.539\,y - 3z)$ v/m



Example :-

An EMW travels in free space with field $= 100\, e^{j(0.866y + .5z)} \, \hat{a}_x \, \frac{V}{m}$

Determine

(a) ω and λ

(b) the other component of field and determine the polarization

(c) The time average power